Fundamentals of Normal Metal and Superconductor Electrodynamics

Steven M. Anlage

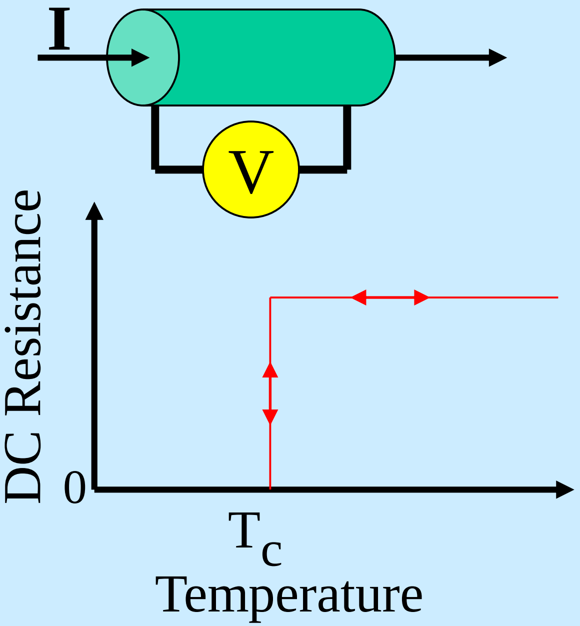
Center for Nanophysics and Advanced Materials
Physics Department
University of Maryland
College Park, MD 20742-4111 USA
anlage@umd.edu

Outline

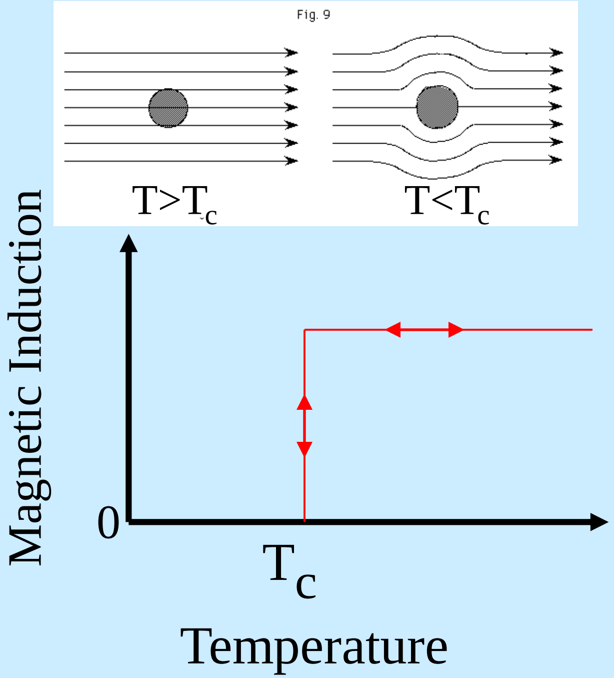
- High Frequency Electrodynamics of Superconductors
- Experimental High Frequency Superconductivity
- Further Reading

The Three Hallmarks of Superconductivity

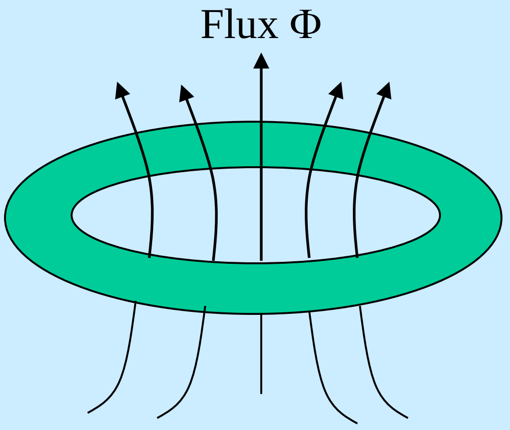
Zero Resistance



Complete Diamagnetism



Macroscopic Quantum Effects



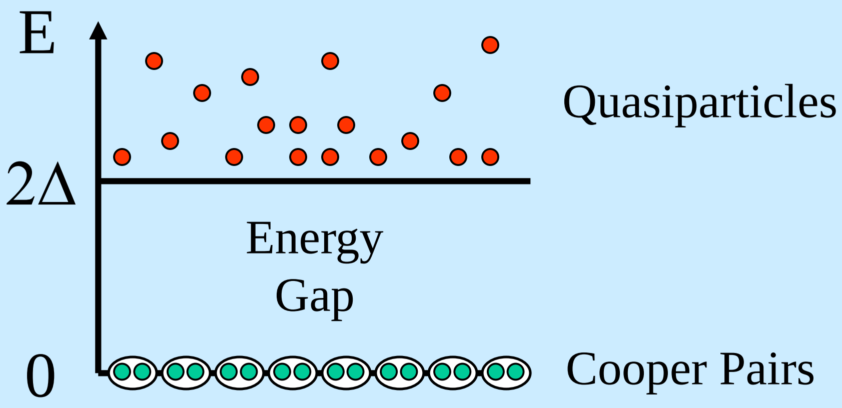
Flux quantization $\Phi = n\Phi_0$

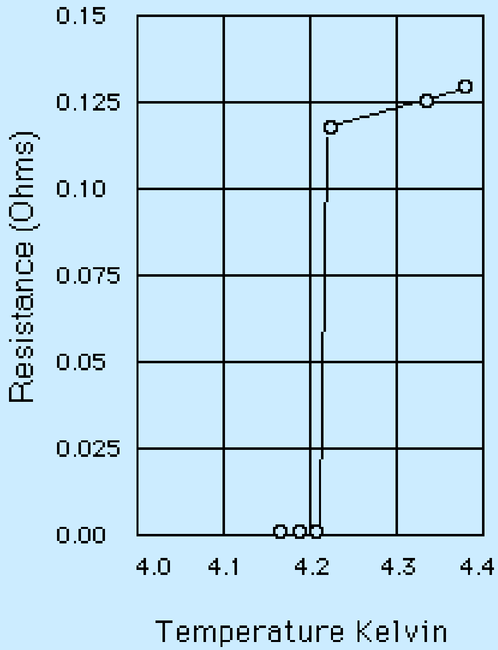
Josephson Effects

Zero Resistance

R = 0 only at ω = 0 (DC)

R > 0 for ω > 0

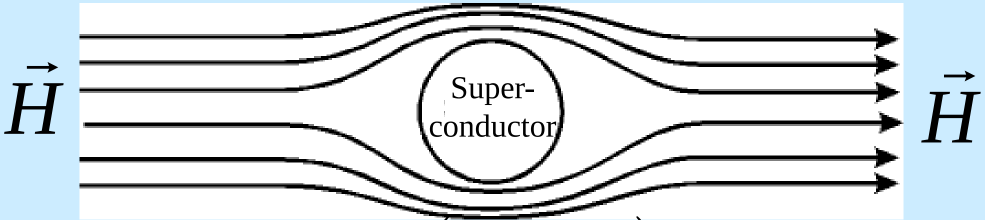




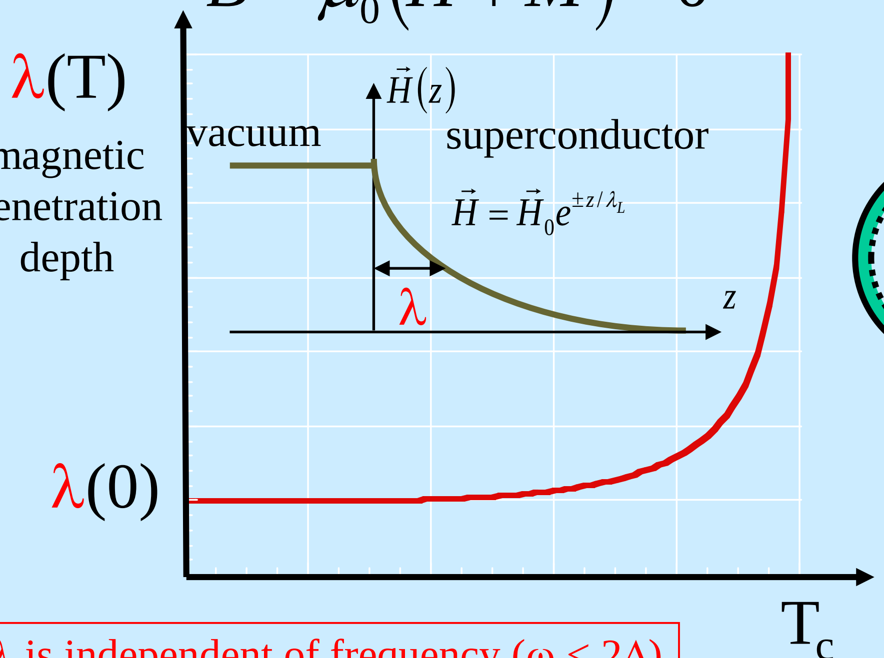
The Kamerlingh Onnes resistance measurement of mercury. At 4.15K the resistance suddenly dropped to zero

Perfect Diamagnetism

Magnetic Fields and Superconductors are not generally compatible

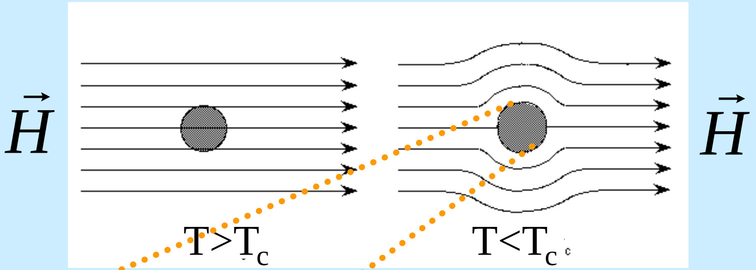


$$\vec{B} = \mu_0\left(\vec{H} + \vec{M}\right) = 0$$

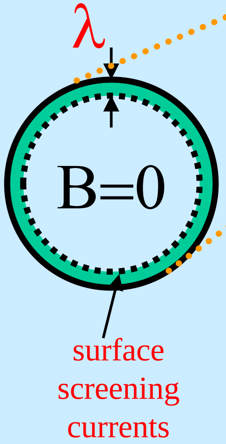


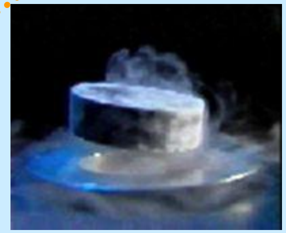
λ is independent of frequency ($\omega < 2\Delta$)

The Meissner Effect



Spontaneous exclusion of magnetic flux





The Yamanashi MLX01 MagLev test vehicle achieved a speed of 361 mph (581 kph) in 2003

High Frequency Electrodynamics of Superconductors

- **Why are Superconductors so Useful at High Frequencies?**
- **Normal Metal Electrodynamics**
- **The Two-Fluid Model**
- **London Equations**
- **BCS Electrodynamics**
- **Nonlinear Surface Impedance**

Why are Superconductors so Useful at High Frequencies?

<u>Low Losses</u>:

Filters have low insertion loss ➔ Better S/N, filters can be made small

High Q ➔ Filters have steep skirts, good out-of-band rejection

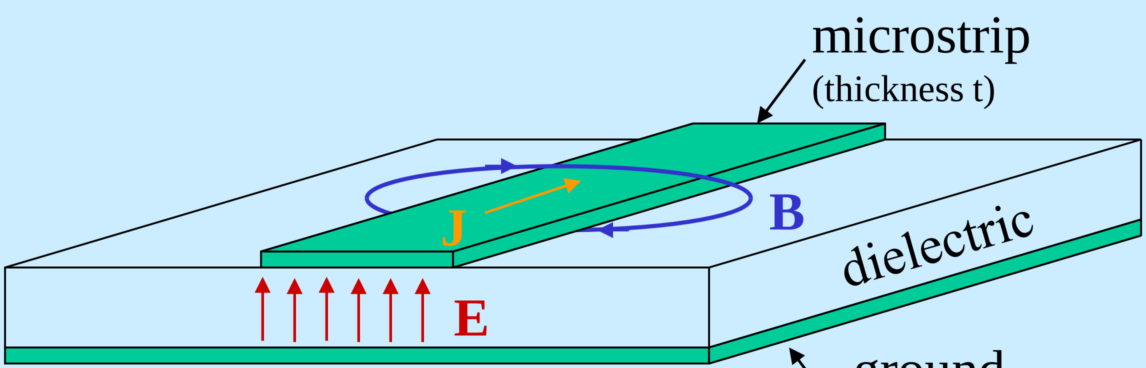
NMR/MRI SC RF pickup coils ➔ x10 improvement in speed of spectrometer

<u>Low Dispersion</u>:

SC transmission lines can carry short pulses with little distortion

RSFQ logic pulses – 1 ps long, ~2 mV in amplitude: $\int V(t)\,dt = \Phi_0 = 2.07\ \mathrm{mV \cdot ps}$

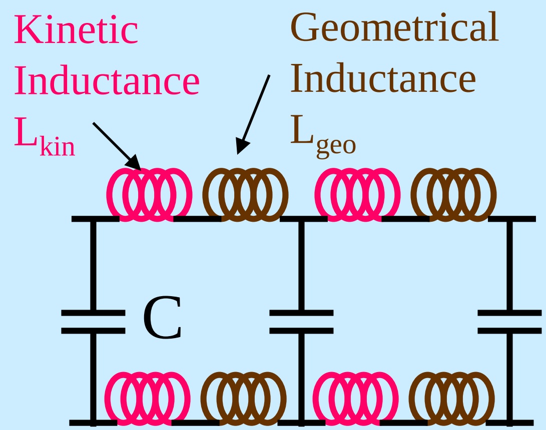
Superconducting Transmission Lines



propagating TEM wave

attenuation α ~ 0

$$E,\ B \sim e^{-\alpha z}$$

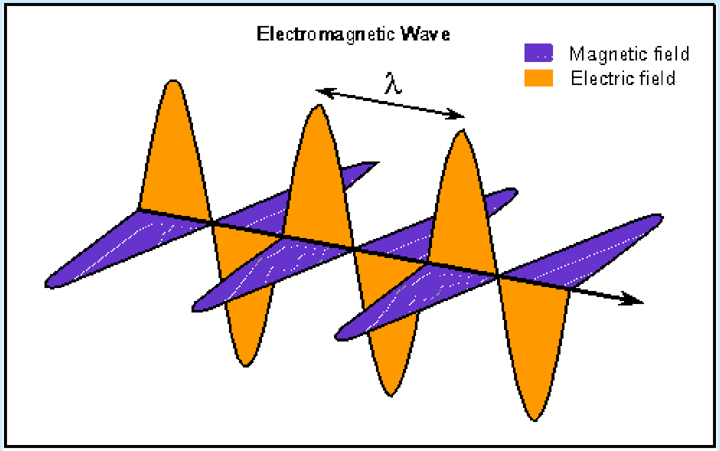


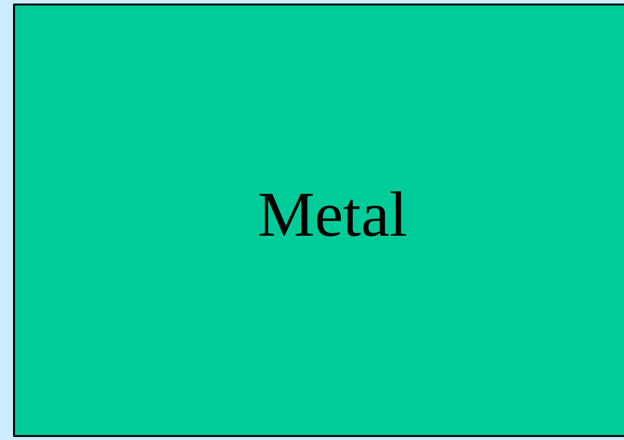
$$L_{kin} \sim \frac{\lambda^2}{t} \qquad v_{phase} = \frac{1}{\sqrt{LC}}$$

$L = L_{kin} + L_{geo}$ is frequency independent

Normal Metal Electrodynamics

Consider a TEM wave incident normally on a metal half-space





Continuity Equation

$$\vec{\nabla} \bullet \vec{J}_{Free} = -\frac{\partial \rho_{Free}}{\partial t}$$

$$\vec{\nabla} \bullet \left(\sigma \vec{E}\right) = -\frac{\partial \rho_{Free}}{\partial t}$$

$$\frac{\sigma \rho_{Free}}{\varepsilon} = -\frac{\partial \rho_{Free}}{\partial t}$$

So $\rho_{Free}(t) = \rho_{Free}(0)\, e^{-(\sigma/\varepsilon)t}$

Constitutive equations for metal

$\vec{J}_{Free} = \sigma \vec{E}$ Ohm's law (local limit)

$\vec{D} = \varepsilon \vec{E}$ $\vec{B} = \mu \vec{H}$ LIH media

$\tau = \rho\varepsilon$ ~ (1 μΩcm)(8.85 x 10^{-12} F/m)

~ 10^{-19} s

Hence we can ignore free charge in the conductor

In reality free charge dissipates at the collision time scale, $\tau_c \sim 10^{-14} - 10^{-12}$ s

Normal Metal Electrodynamics

Take the curl of the curl equations

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = -\frac{\partial}{\partial t}\vec{\nabla}\times\vec{B} \qquad \vec{\nabla}\times\vec{\nabla}\times\vec{B} = \mu\sigma\vec{\nabla}\times\vec{E} + \mu\varepsilon\frac{\partial}{\partial t}\vec{\nabla}\times\vec{E}$$

$$\nabla^2\vec{E} = \mu\sigma\frac{\partial\vec{E}}{\partial t} + \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} \qquad \nabla^2\vec{B} = \mu\sigma\frac{\partial\vec{B}}{\partial t} + \mu\varepsilon\frac{\partial^2\vec{B}}{\partial t^2}$$

These are wave equations with a $\mu\sigma$ dissipative term

Ansatz $\tilde{\vec{E}} = \tilde{\vec{E}}_0\, e^{i\left(\tilde{k}z-\omega t\right)}$ $\qquad \tilde{\vec{B}} = \tilde{\vec{B}}_0\, e^{i\left(\tilde{k}z-\omega t\right)}$

with $\tilde{k} = k + i\kappa$

One finds $k = \kappa \cong \sqrt{\frac{\sigma\omega\mu}{2}}$ The waves oscillate and decay as they enter the metal

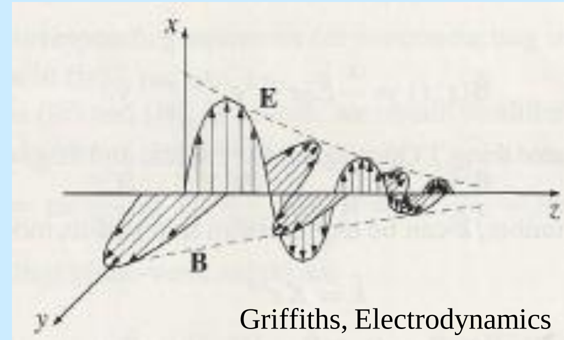
Define the skin depth $\delta = \frac{1}{\kappa} = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{2\rho}{\omega\mu}}$

For a metal with $\rho = 1\ \mu\Omega$-cm at 2.5 GHz, $\delta = 1.0\ \mu m$

Normal Metal Electrodynamics

$$\tilde{\vec{E}} = \tilde{E}_0 \, e^{i(z/\delta - \omega t)} e^{-z/\delta} \, \hat{x}$$

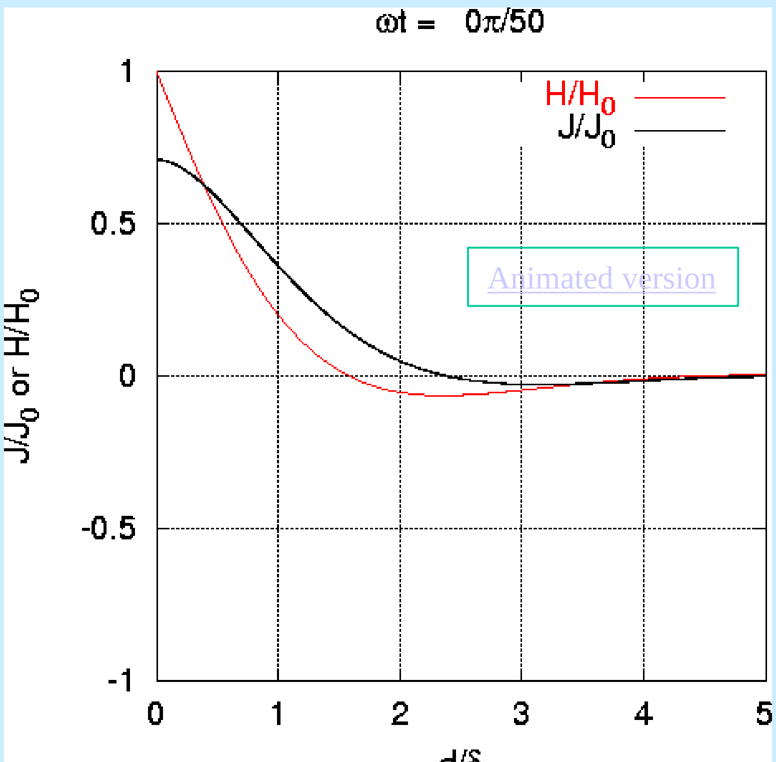
$$\tilde{\vec{B}} = \frac{\tilde{k}}{\omega} \tilde{E}_0 \, e^{i(z/\delta - \omega t)} e^{-z/\delta} \, \hat{y}$$

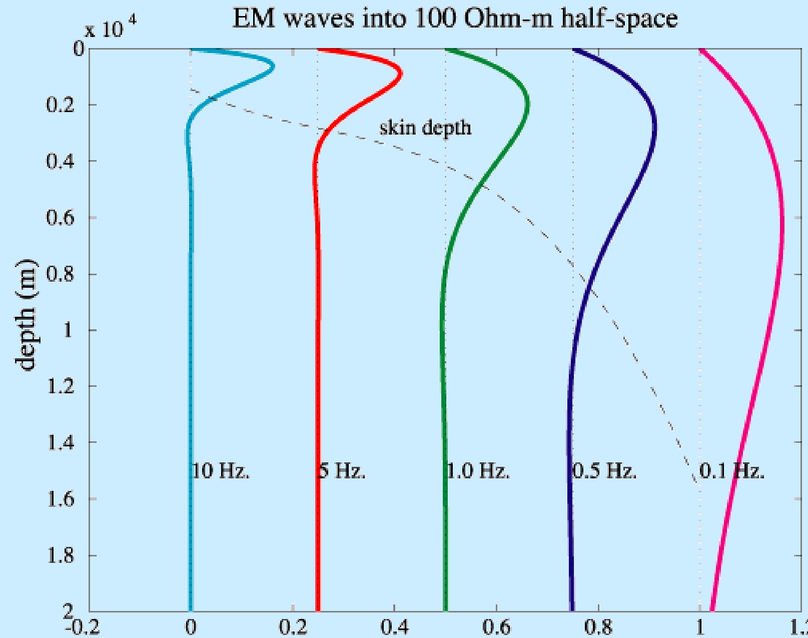


Griffiths, Electrodynamics

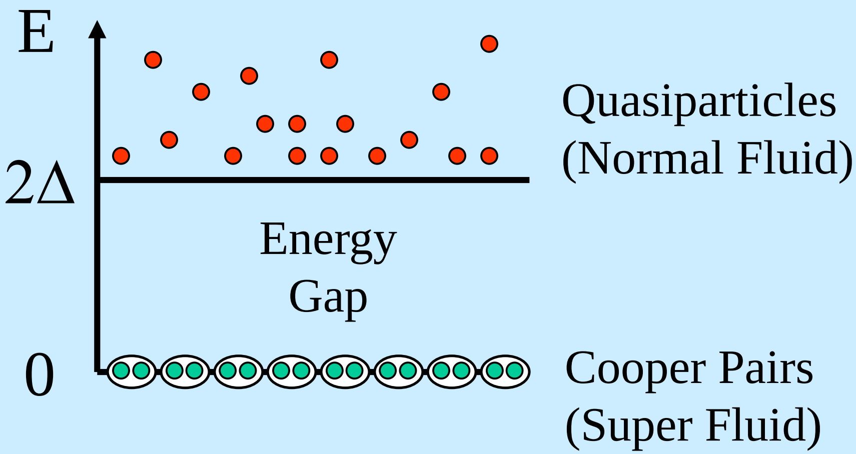
Phase difference between E, B:

$$\phi = \tan^{-1}(\kappa / k)$$

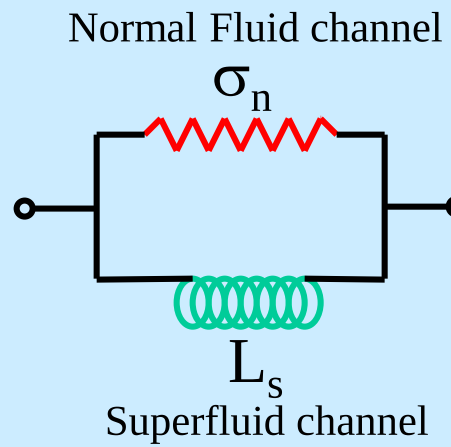


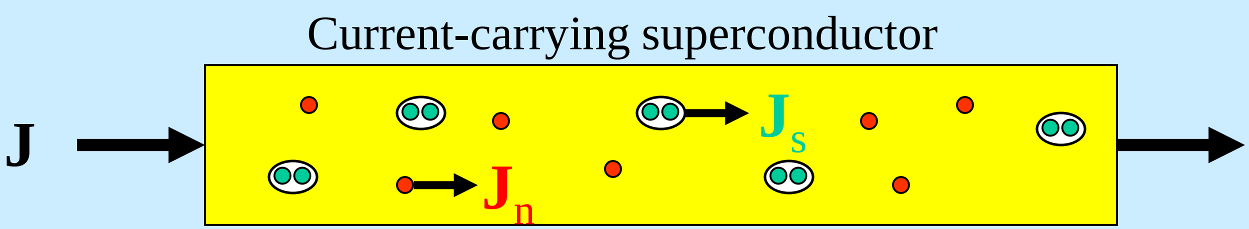


Electrodynamics of Superconductors in the Meissner State



≈





$\mathbf{J} = \mathbf{J}_s + \mathbf{J}_n$

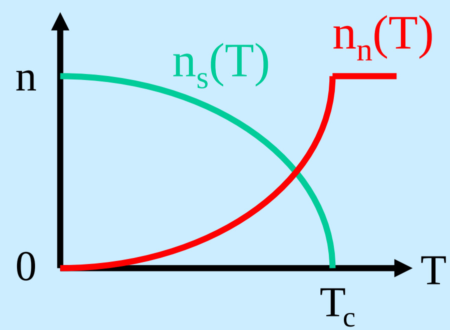
$\mathbf{n} = \mathbf{n}_s(\mathbf{T}) + \mathbf{n}_n(\mathbf{T})$

$J = \sigma E$

$\sigma = \sigma_n - i\,\sigma_2$

$\sigma_n = n_n e^2 \tau / m$

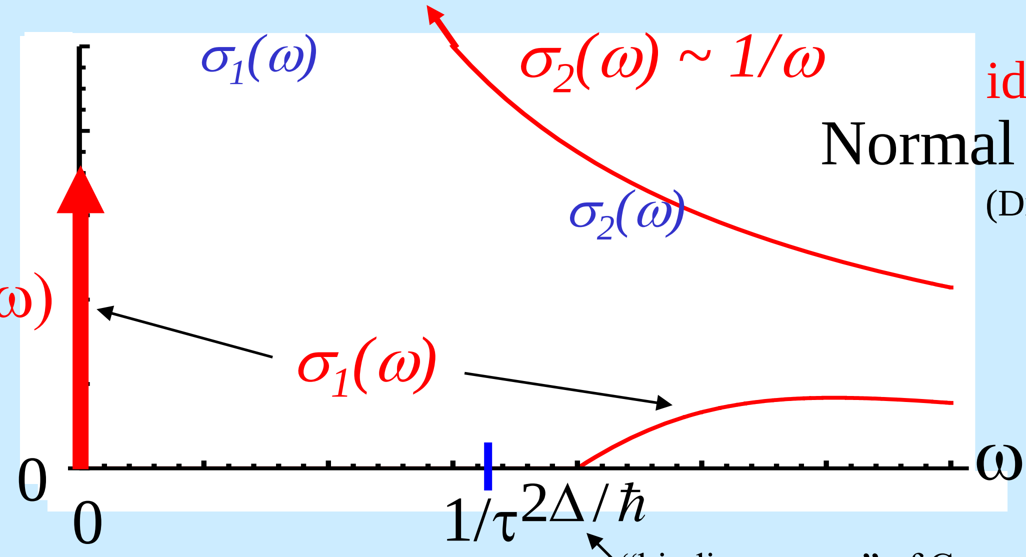
$\sigma_2 = n_s e^2 / m\omega$



n_n = number of QPs
n_s = number of SC electrons
τ = QP momentum relaxation time
m = carrier mass
ω = frequency

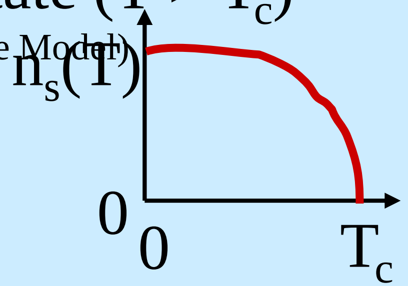
Superconductor Electrodynamics

$\sigma = \sigma_1 - i\sigma_2$



Superfluid density
$\lambda^2 \sim m/n_s$
$\sim 1/\omega_{ps}^2$

T = 0
ideal s-wave
Normal State (T > T_c)
(Drude Model)



"binding energy" of Cooper pair (100 GHz ~ few THz)

Surface Impedance (ω > 0) $Z_s = R_s + iX_s = \sqrt{i\omega\mu_0/\sigma}$

Normal State

$$R_s = X_s \cong \sqrt{\frac{\omega\mu_0}{2\sigma_1}} = \frac{1}{\sigma_1\delta}$$

Superconducting State (ω < 2Δ)

$$R_s \sim \sigma_1 \approx 0 \qquad X_s = \mu_0\omega\lambda$$

Penetration depth
$\lambda(0) \sim 20 - 200$ nm

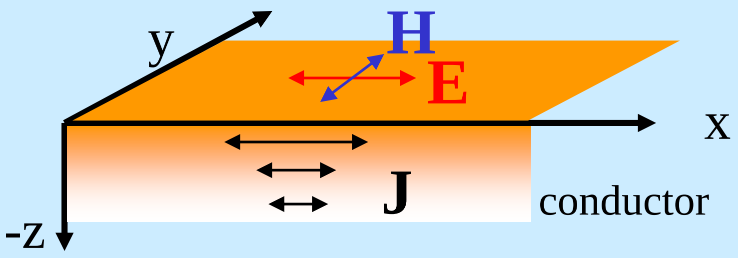
Finite-temperature: $X_s(T) = \omega L = \omega\mu_0\lambda(T) \rightarrow \infty$ as $T \rightarrow T_c$ (and $\omega_{ps}(T) \rightarrow 0$)

Narrow wire or thin film of thickness t : $L(T) = \mu_0\lambda(T)\coth(t/\lambda(T)) \rightarrow \mu_0\lambda^2(T)/t$
Kinetic Inductance

Surface Impedance

$$Z_s = R_s + iX_s = \frac{\left|\vec{E}_\parallel\right|}{\int \vec{J}_\parallel(z)\,dz} = \sqrt{\frac{i\omega\mu}{\sigma}}$$

Local Limit



Surface Resistance R_s: Measure of Ohmic power dissipation

$$P_{Dissipated} = \frac{1}{2}\mathrm{Re}\left\{\iiint_{Volume} \vec{J}\cdot\vec{E}\,dV\right\} = \frac{1}{2}R_s \iint_{Surface}\left|\vec{H}\right|^2 dA \sim \frac{1}{2}I^2 R_s$$

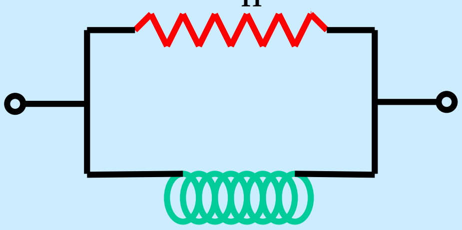
Surface Reactance X_s: Measure of stored energy per period

$$W_{Stored} = \frac{1}{2}\iiint_{Volume}\left(\mu\left|\vec{H}\right|^2 + \mathrm{Im}\left\{\vec{J}\cdot\vec{E}\right\}\right)dV = \frac{1}{2\omega}X_s \iint_{Surface}\left|\vec{H}\right|^2 dA \sim \frac{1}{2}LI^2$$

L_{geo} $L_{kinetic}$

$$X_s = \omega L_s = \omega\mu\lambda$$

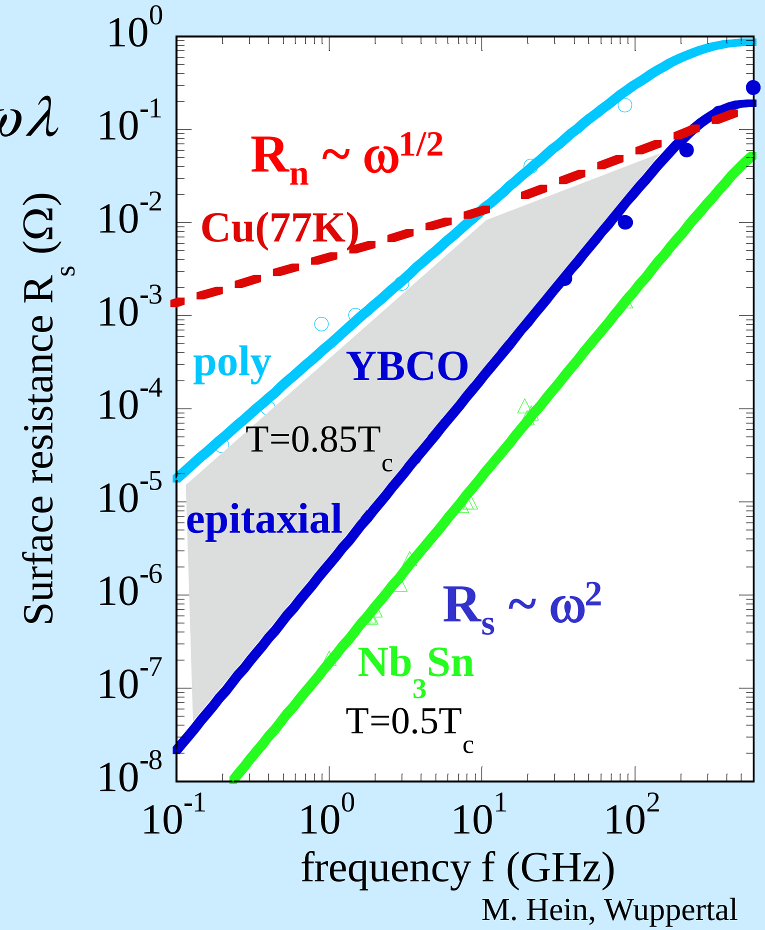
Two-Fluid Surface Impedance



$$Z_s = R_s + iX_s$$

$$R_s = \frac{1}{2}\omega^2 \mu_0 \lambda^3 \sigma_n$$

$$X_s = \mu_0 \omega \lambda$$



Because $R_s \sim \omega^2$:
The advantage of HTS over Cu diminishes with increasing frequency

R_s crossover at f ~ 100 GHz at 77 K

$R_s \sim \sigma_n$ $\qquad$ $R_n \sim 1/\sqrt{\sigma_n}$

The London Equations

Newton's 2nd Law for a charge carrier

$$m\frac{d\vec{v}}{dt} = e\vec{E} - \frac{m\vec{v}}{\tau}$$

τ = momentum relaxation time

$\mathbf{J}_s = n_s\, e\, \mathbf{v}_s$

Superconductor:
$1/\tau \rightarrow 0$

$$\frac{d\vec{J}_s}{dt} = \frac{n_s e^2}{m}\vec{E} = \frac{1}{\mu_0 \lambda_L^2}\vec{E}$$

1st London Equation

1st London Eq. and $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (Faraday) yield:

$$\frac{d}{dt}\left[\nabla \times \vec{J}_s + \frac{n_s e^2}{m}\vec{B}\right] = 0$$

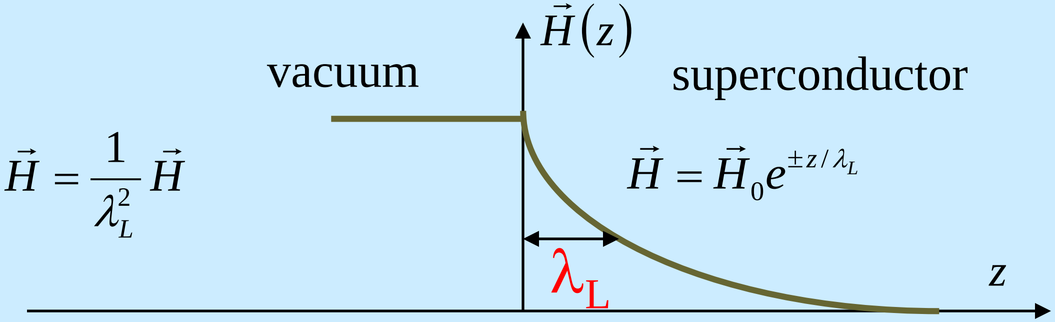
London surmise ➔

$$\nabla \times \vec{J}_s + \frac{n_s e^2}{m}\vec{B} = 0$$

2nd London Equation

These equations yield the Meissner screening

$$\nabla^2 \vec{H} = \frac{1}{\lambda_L^2}\vec{H}$$



$$\lambda_L \equiv \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

λ_L is frequency independent ($\omega < 2\Delta$)

$\lambda_L \sim 20 - 200$ nm

The London Equations continued

	Normal metal	Superconductor	
E is the source of J_n	$\vec{J}_n = \sigma_n \vec{E}$	$\frac{d\vec{J}_s}{dt} = \frac{1}{\mu_0 \lambda_L^2}\vec{E}$	**E**=0: J_s goes on forever
Lenz's Law	$\frac{d}{dt}\left[\nabla \times \vec{J}_n + \frac{1}{\mu_0 \lambda_L^2}\vec{B}\right] = 0$	$\mu_0 \lambda_L^2 \left(\nabla \times \vec{J}_s\right) = -\vec{B}$	**B** is the source of J_s, spontaneous flux exclusion

1st London Equation ➔ **E** is required to maintain an ac current in a SC
Cooper pair has finite inertia ➔ QPs are accelerated and dissipation occurs

BCS Microwave Electrodynamics

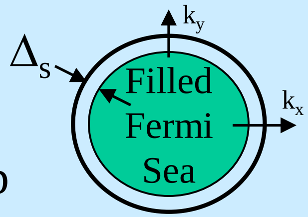
Low Microwave Dissipation

Full energy gap → R_s can be made arbitrarily small

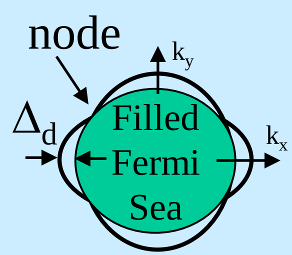
$$R_s \approx e^{-\Delta(0)/k_B T}$$

for T < T_c/3 in a fully-gapped SC

$$R_s = R_{BCS}(T) + R_{s,\mathrm{residual}}$$



$R_{s,\mathrm{residual}}$ ~ 10^{-9} Ω at 1.5 GHz in Nb

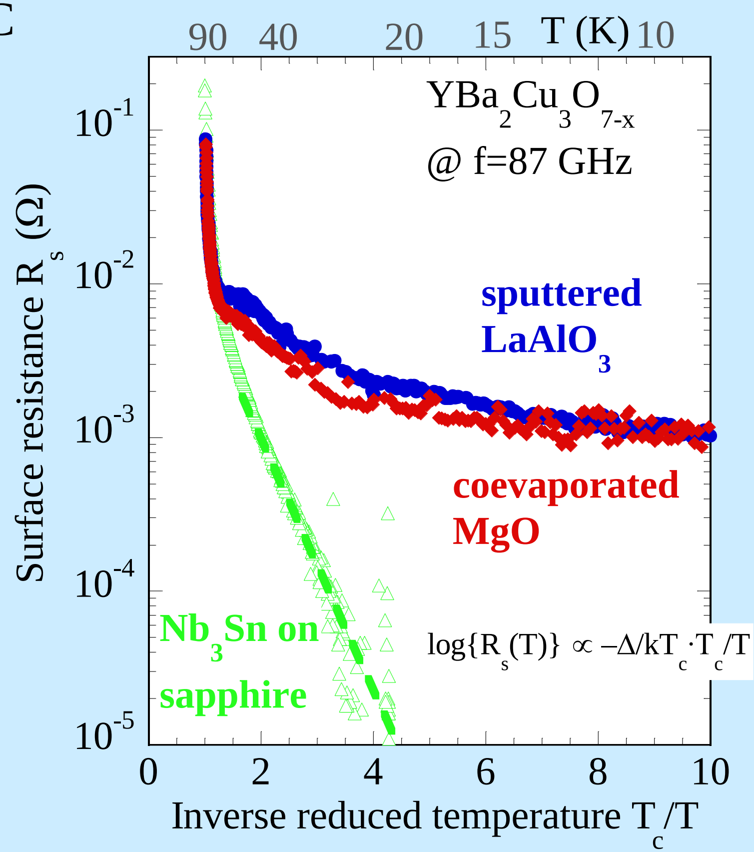


HTS materials have nodes in the energy gap. This leads to power-law behavior of λ(T) and R_s(T) and residual losses

$$\lambda(T) = \lambda(0) + a\,T$$

$$R_s = R_{s,\mathrm{residual}} + b\,T$$

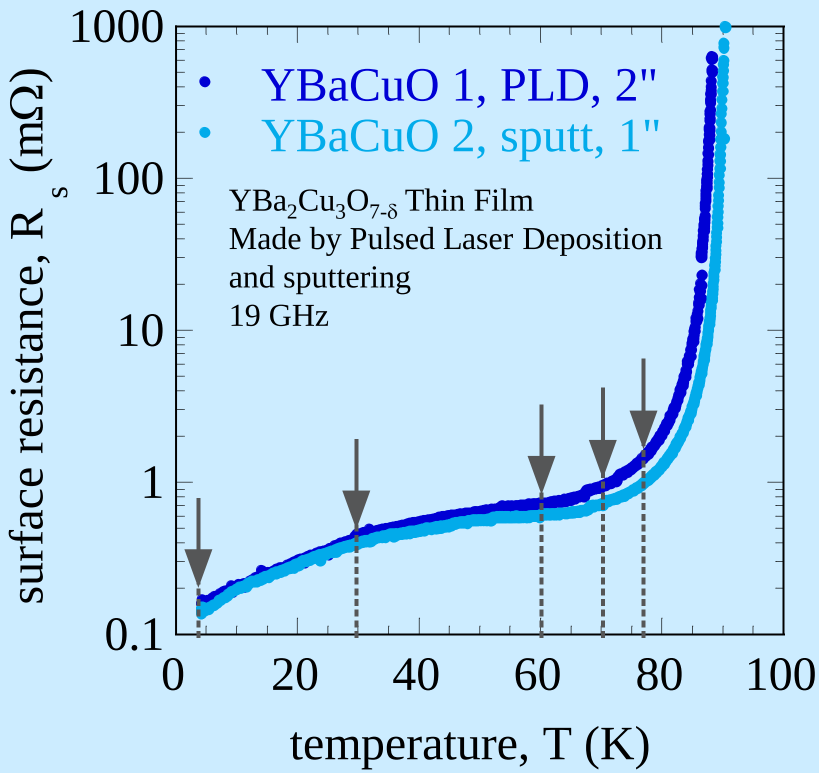
$R_{s,\mathrm{residual}}$ ~ 10^{-5} Ω at 10 GHz in $YBa_2Cu_3O_{7-\delta}$

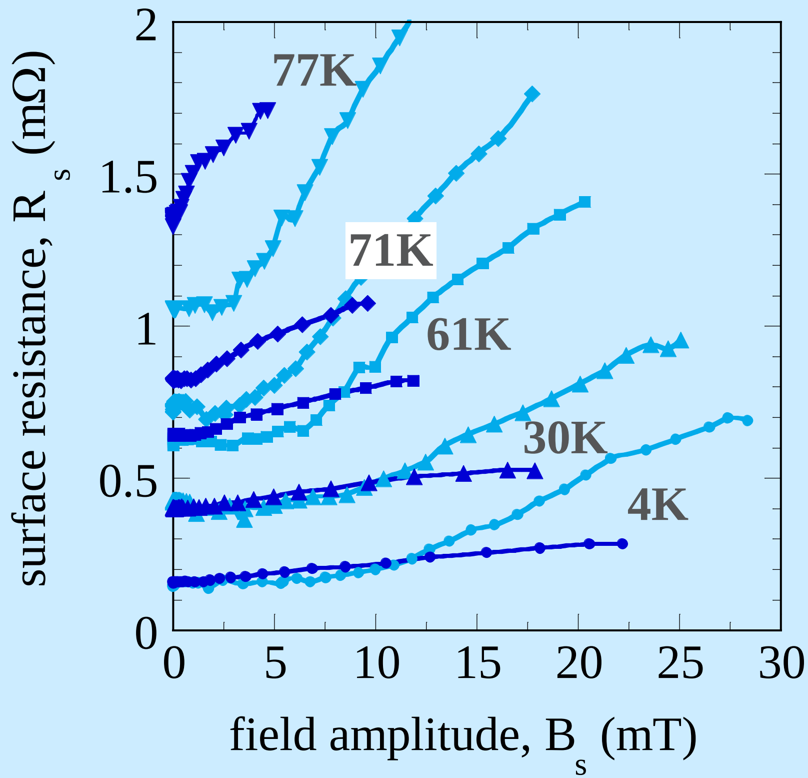


M. Hein, Wuppertal

Nonlinear Surface Impedance of Superconductors

The surface resistance and reactance values depend on the rf current level flowing in the superconductor





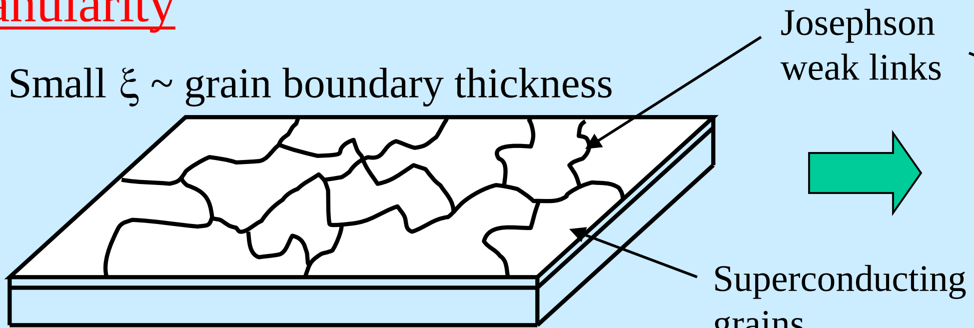
Similar results for $X_s(B_s)$

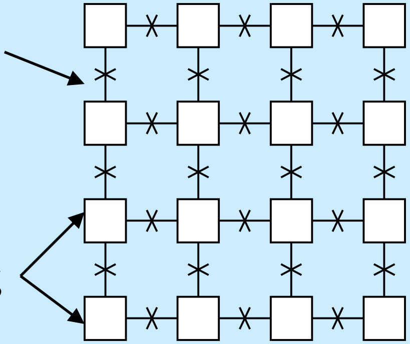
Data from M. Hein, Wuppertal

How can Superconductors become Nonlinear?

Granularity

Small ξ ~ grain boundary thickness





JJs have a strongly nonlinear impedance

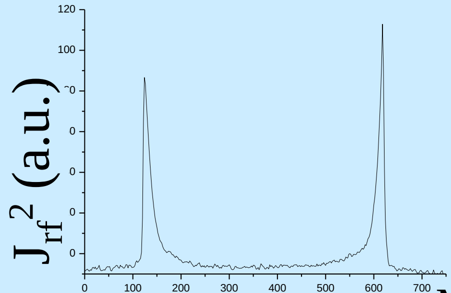
McDonald + Clem
PRB 56, 14 723 (1997)

$$L_J = \frac{\Phi_0}{2\pi I_c \cos(\delta)}$$

Edge-Current Buildup

+ Vortex Entry and Flow

Heating



Microstrip (Longitudinal view)

Scanning Laser Microscope image
YBCO strip at T = 79 K
f = 5.285 GHz, Laser Spot Size = 1 μm

See Appl. Phys. Lett. **88**, 212503 (2006)

Intrinsic Nonlinear Meissner Effect

rf currents cause de-pairing – convert superfluid into normal fluid

$$\left(\frac{\lambda(0,T)}{\lambda(J,T)}\right)^2 = 1 - \left(\frac{J}{J_{NL}(T)}\right)^2$$

$J_{NL}(T)$ calculated by theory (Dahm+Scalapino)

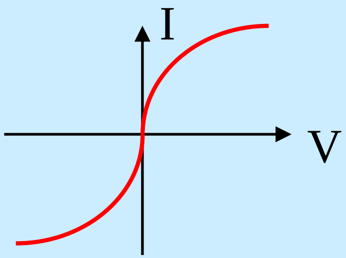
Nonlinearities are generally strongest near T_c and weaken at lower temperatures

How to Model Superconducting Nonlinearity?

(1) Taylor series expansion of nonlinear I-V curve (Z. Y. Shen)

= 0 if I(-V) = -I(V)

$$I(V) = I(0) + \left(\frac{dI}{dV}\right)_{V=0} \delta V + \frac{1}{2!}\left(\frac{d^2 I}{dV^2}\right)_{V=0} \delta V^2 + \frac{1}{3!}\left(\frac{d^3 I}{dV^3}\right)_{V=0} \delta V^3 + O\left(\delta V^4\right)$$

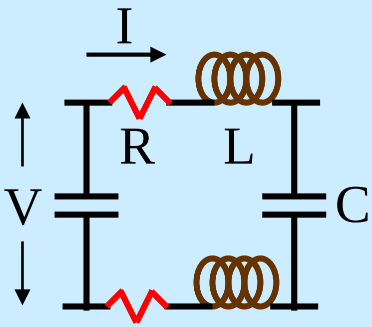


1/R linear term

3rd order term dominates

V = V_0 sin(ωt) input yields ~ V_0^3 sin(3ωt) + … output

(2) Nonlinear transmission line model (Dahm and Scalapino)



$$\frac{\partial I}{\partial z} = -C\frac{\partial V}{\partial t}$$

$$\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t} - RI$$

3rd harmonics and 3rd order IMD result

$$\frac{\partial^2 I}{\partial z^2} = LC\frac{\partial^2 I}{\partial t^2} + RC\frac{\partial I}{\partial t} + C\frac{\partial L}{\partial t}\frac{\partial I}{\partial t} + C\frac{\partial R}{\partial t}I$$

additional terms

L and R are nonlinear:

$$L = L_0 + \Delta L\left(\frac{I}{I_{NL}}\right)^2 \qquad R = R_0 + \Delta R\left(\frac{I}{I_{NL}}\right)^2$$

Experimental High Frequency Superconductivity

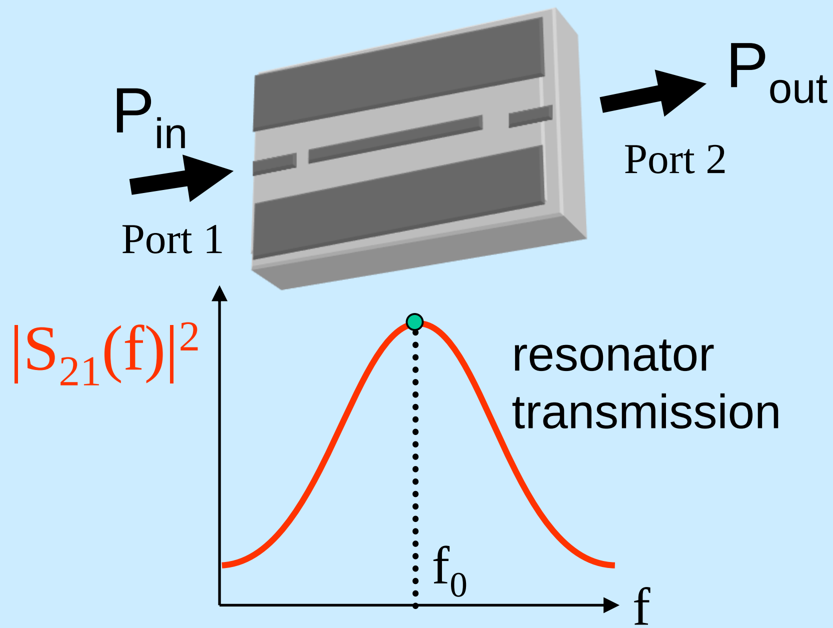
- **Resonators**
- **Cavity Perturbation**
- **Measurements of Nonlinearity**
- **Topics of Current Interest**
- **Microwave Microscopy**

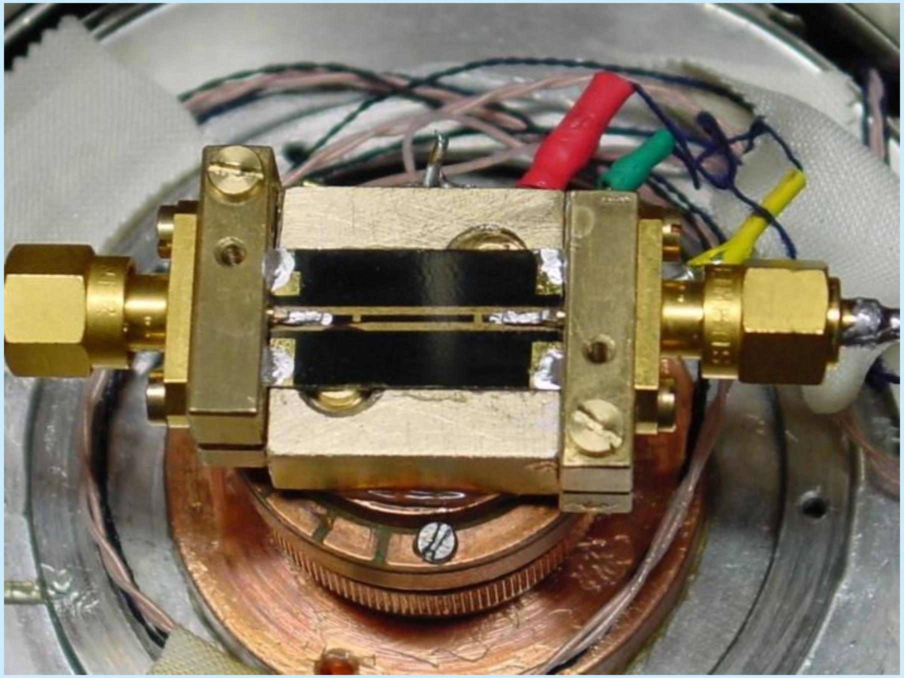
Resonators

… the building block of superconducting applications …

Microwave surface impedance measurements
Cavity Quantum Electrodynamics of Qubits
Superconducting RF Accelerators
Metamaterials ($\mu_{eff} < 0$ ‘atoms’)
etc.

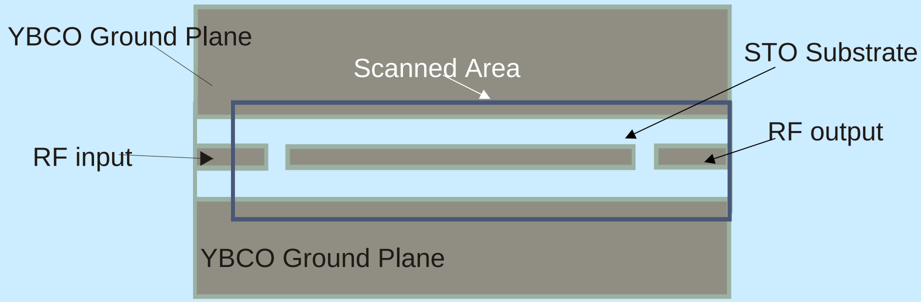
co-planar waveguide resonator

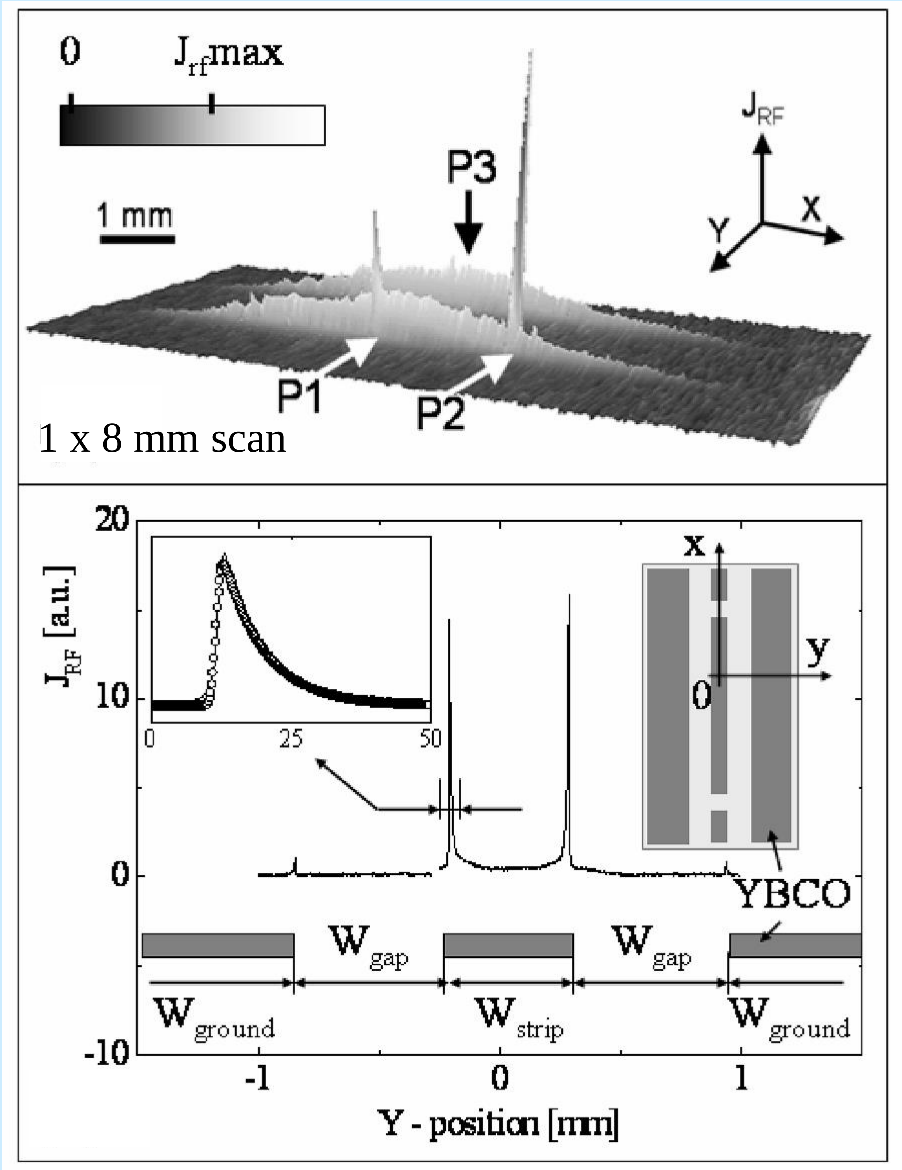




Resonators (continued)

YBCO/$LaAlO_3$
CPW Resonator
Excited in Fundamental Mode
Imaged by Laser Scanning Microscopy*



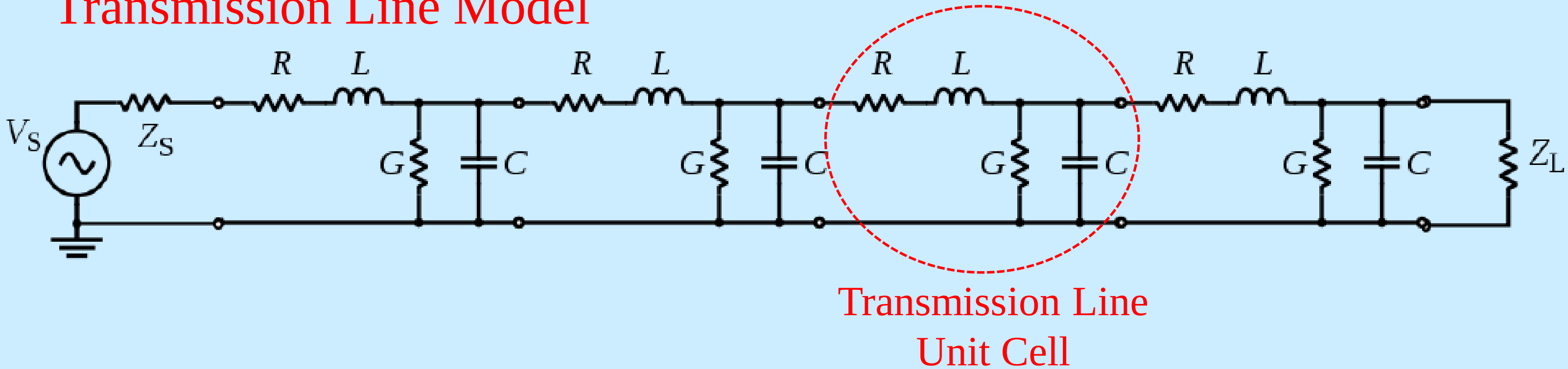


T = 79 K
P = - 10 dBm
f = 5.285 GHz
W_{strip} = 500 μm

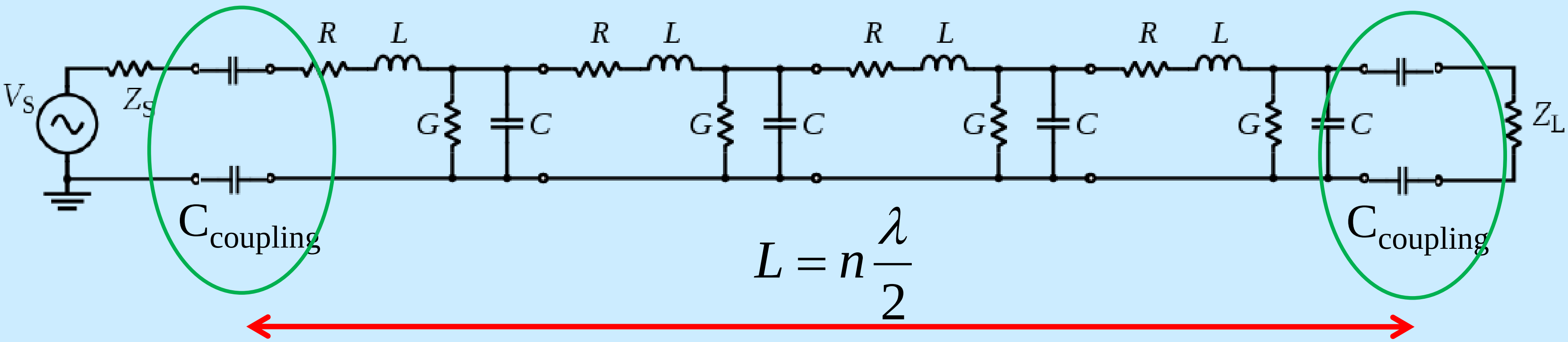
*A. P. Zhuravel, *et al.*, J. Supercond. 19, 625 (2006)

Transmission Line Resonators

Transmission Line Model



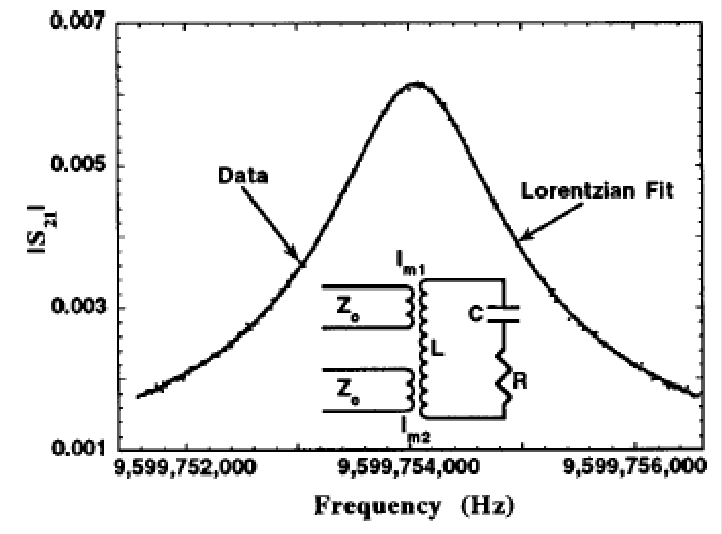
Transmission Line Resonator Model



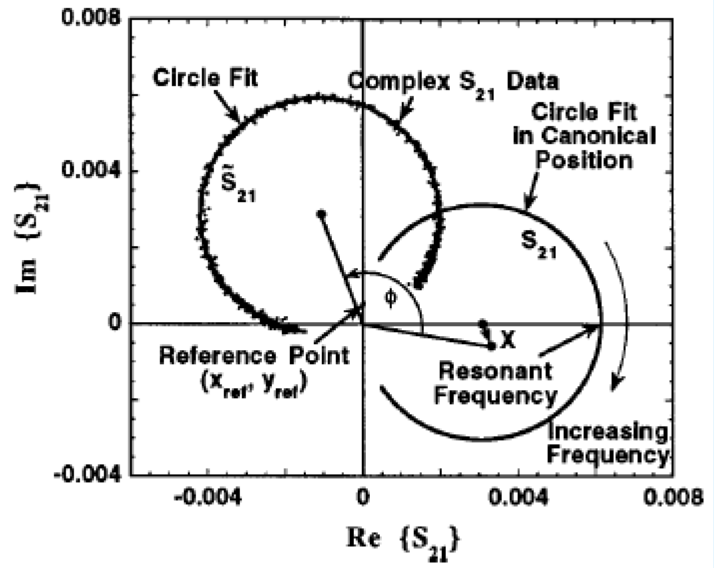
$$L = n\frac{\lambda}{2}$$

$$f_n = n\frac{c}{2L} \qquad n = 1, 2, 3, \ldots$$

Resonators (continued)



$$|S_{21}(f)| = \frac{|\overline{S_{21}}|}{\sqrt{1+4Q^2\left(\frac{f}{f_0}-1\right)^2}}$$



$$S_{21}(f) = \frac{\overline{S_{21}}}{1+i2Q\left(\frac{f}{f_0}-1\right)}$$

$$\tilde{S}_{21} = (S_{21}+X)e^{i\phi}$$

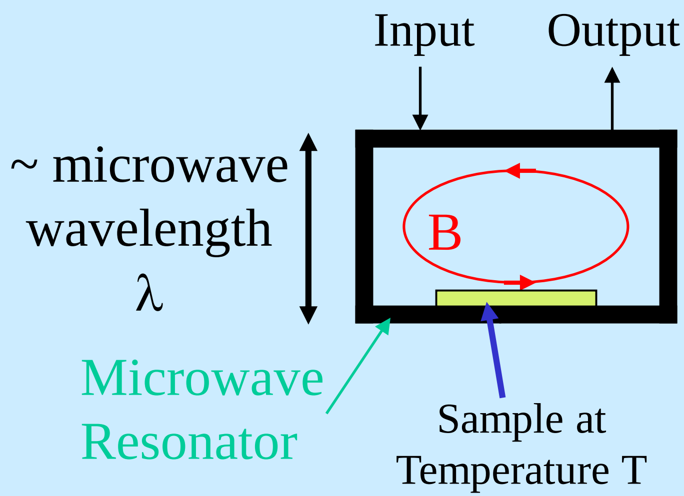
JOURNAL OF APPLIED PHYSICS VOLUME 84, NUMBER 6 3392 15 SEPTEMBER 1998

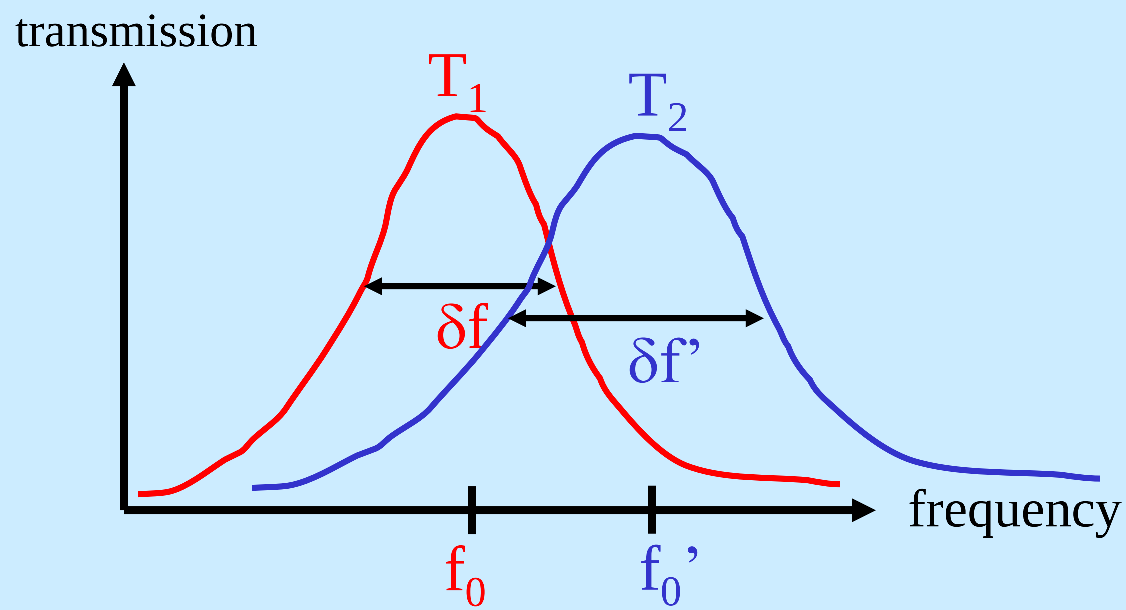
Measurement of resonant frequency and quality factor of microwave resonators: Comparison of methods

Paul J. Petersan and Steven M. Anlage[a)]

Cavity Perturbation

Objective: determine R_s, X_s (or σ_1, σ_2) from f_0 and Q measurements of a resonant cavity containing the sample of interest





Quality Factor

$$Q = \frac{U_{\text{Stored}}}{U_{\text{Dissipated}}} = \frac{f_0}{\delta f}$$

$\Delta f = f_0' - f_0 \propto \Delta$(Stored Energy)

$\Delta(1/2Q) \propto \Delta$(Dissipated Energy)

Cavity perturbation means $\Delta f << f_0$

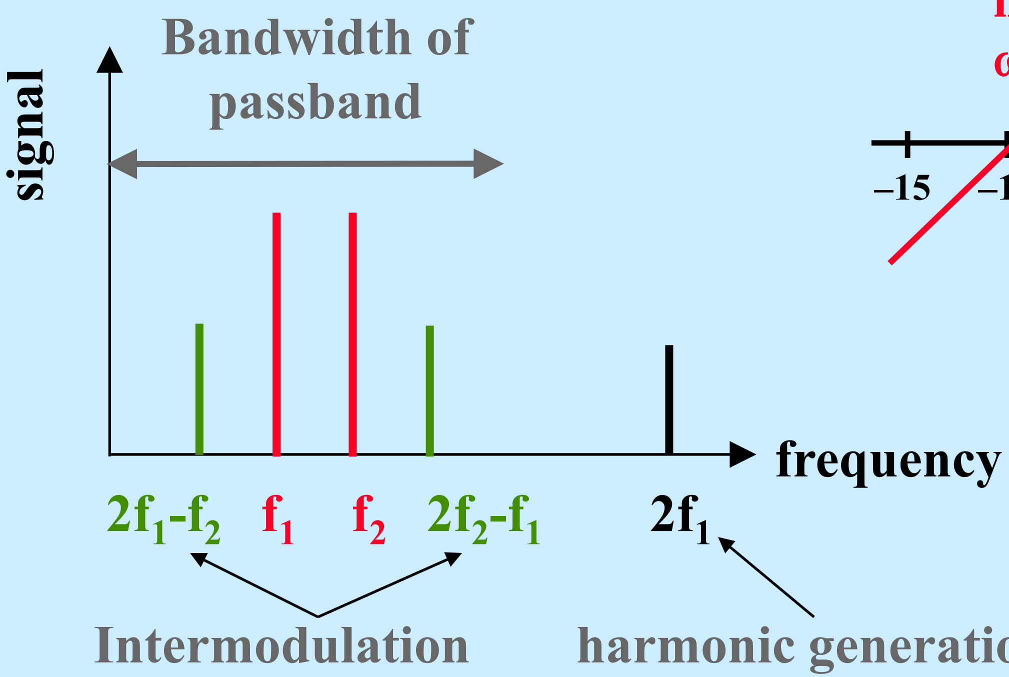
$$R_s = \frac{\Gamma}{Q} \qquad \Delta X_s = \frac{2\Gamma}{\omega}\Delta\omega$$

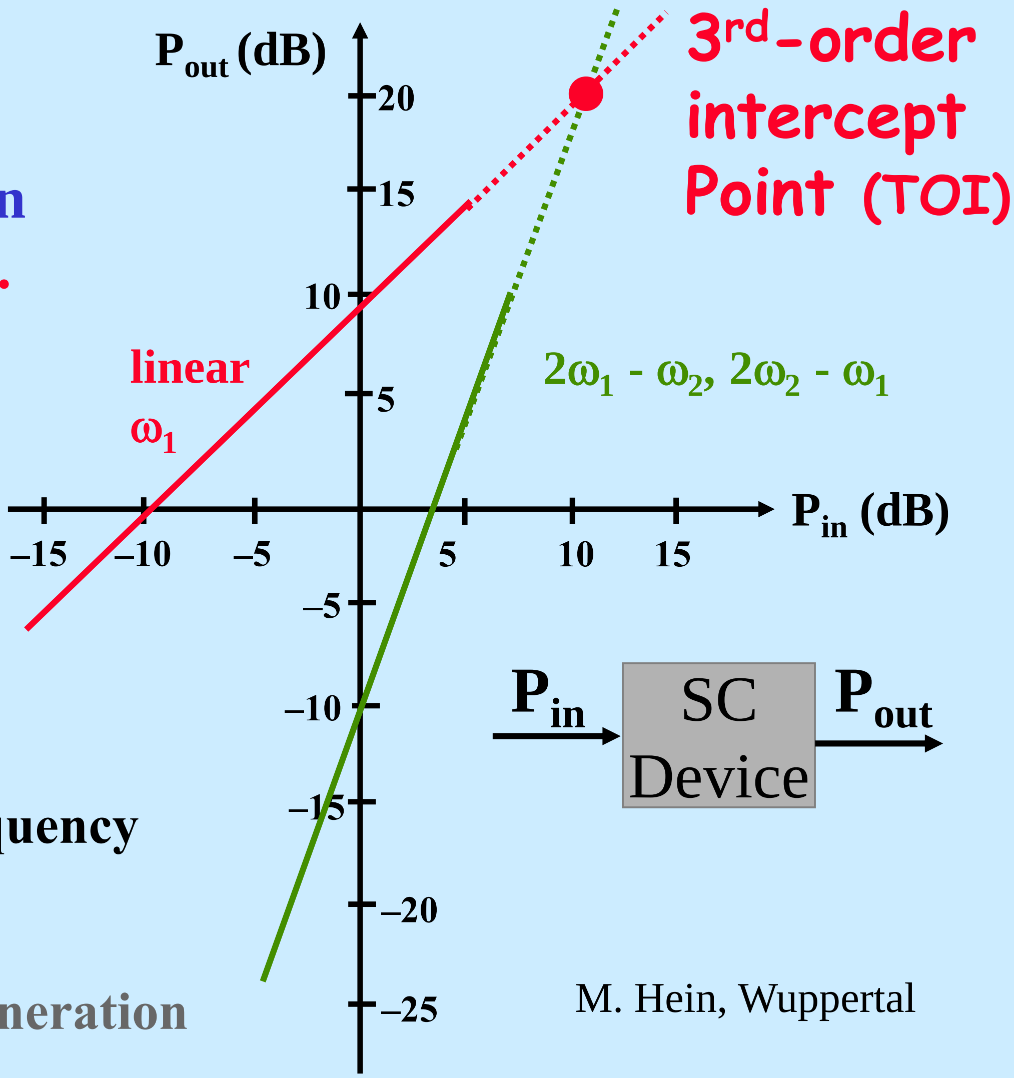
Γ is the sample/cavity geometry factor

Measurement of Nonlinearities

Intermodulation is a practical problem

Nonlinear (i. e., signal strength dependent) microwave response induces undesirable signals within the passband by intermodulation.





M. Hein, Wuppertal

Topics of Current Interest
In Microwave Superconductivity Research

Identifying and eliminating the microscopic sources of extrinsic nonlinearity
- Increase device yield
- Allows further miniaturization of devices
- Allow development of ILC Nb cavities with BCS-limited properties

Superconducting Metamaterials: J. Opt. **13**, 024001 (2011)
- Low-loss, compact, tunable metamaterial ‘atoms’

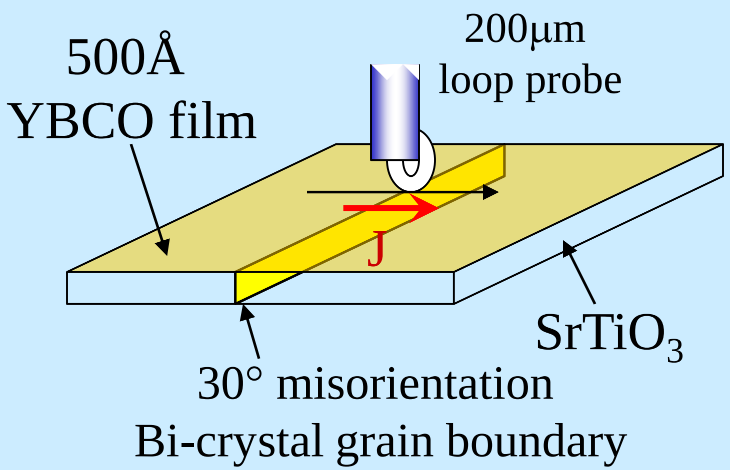
Controlling de-coherence in superconducting qubits
- Identify and eliminate two-level systems in dielectrics

Microwave Microscopy of Superconductors

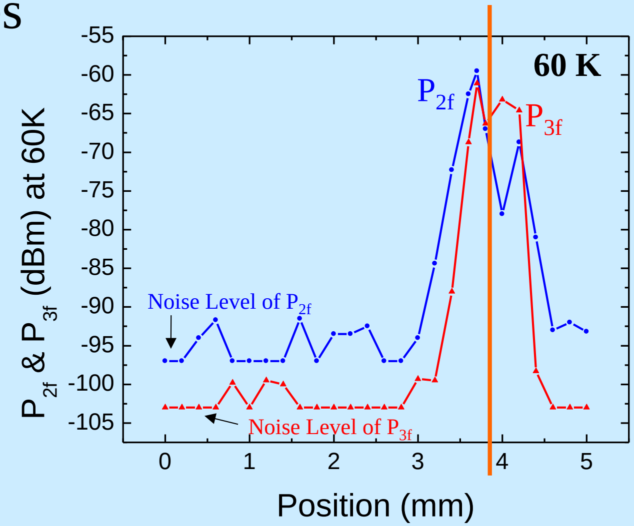
Use near-field optics techniques to obtain super-resolution images of:

1) Materials Properties: Nonlinear response

2) RF fields in operating devices



See:
2MY-08
5MZ-03

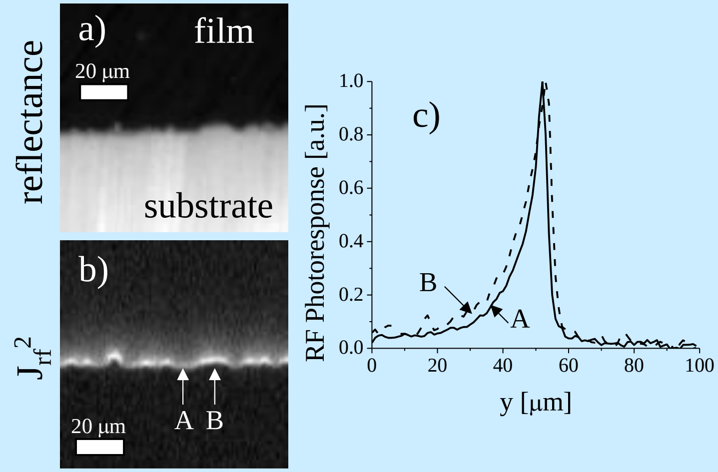


Phys. Rev. B **72**, 024527 (2005)

Laser Scanning Microscopy

Image $J_{rf}^2(x,y)$ in an operating superconducting microwave device

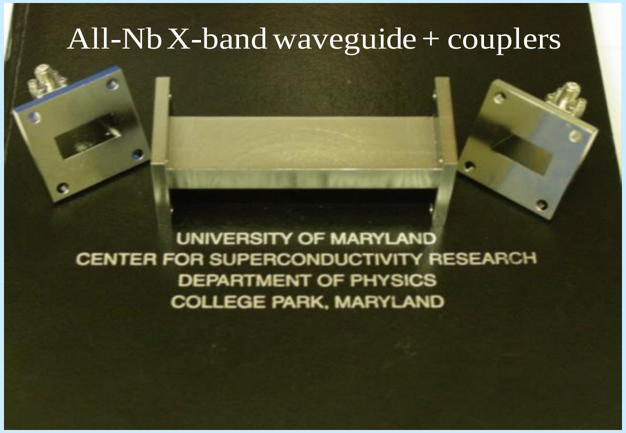
Image J_{IMD}

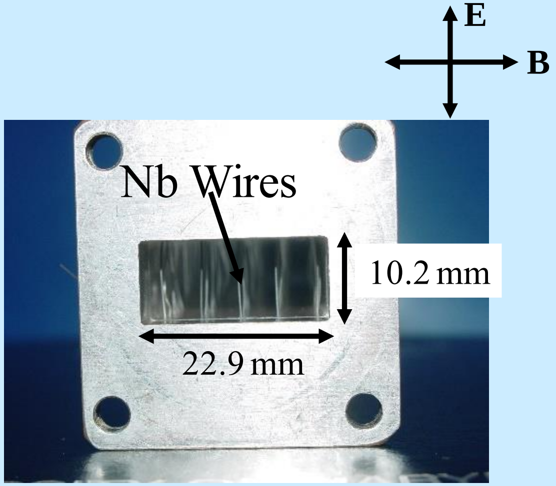


IEEE Trans. Appl. Supercond. **17**, 902 (2007)

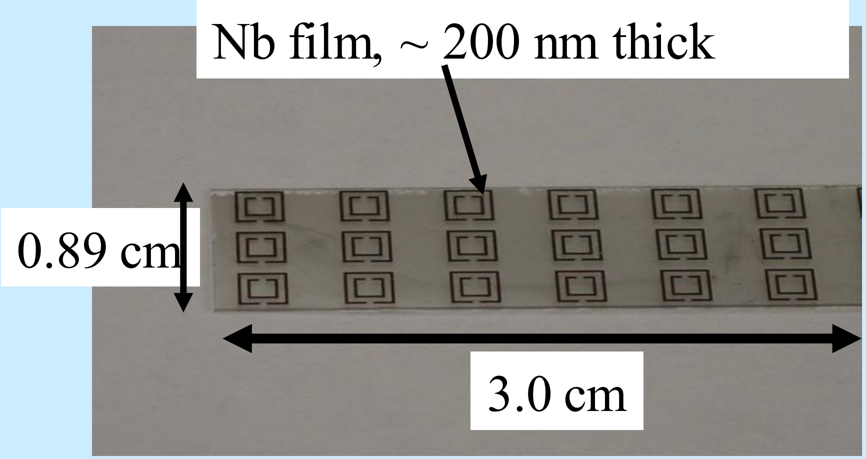
Superconducting Metamaterials

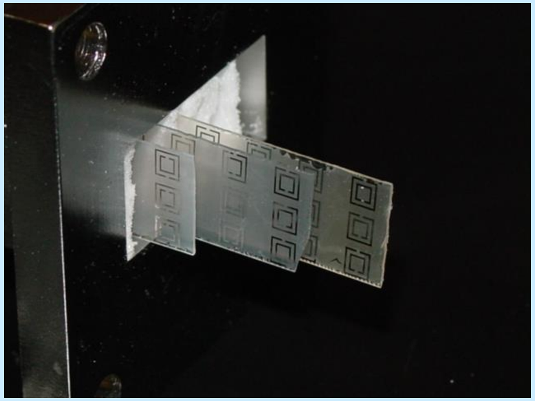
Build artificial 'atoms' with tailored electric and magnetic response
An array of these sub-wavelength 'atoms' are described by ε_{eff}, μ_{eff}



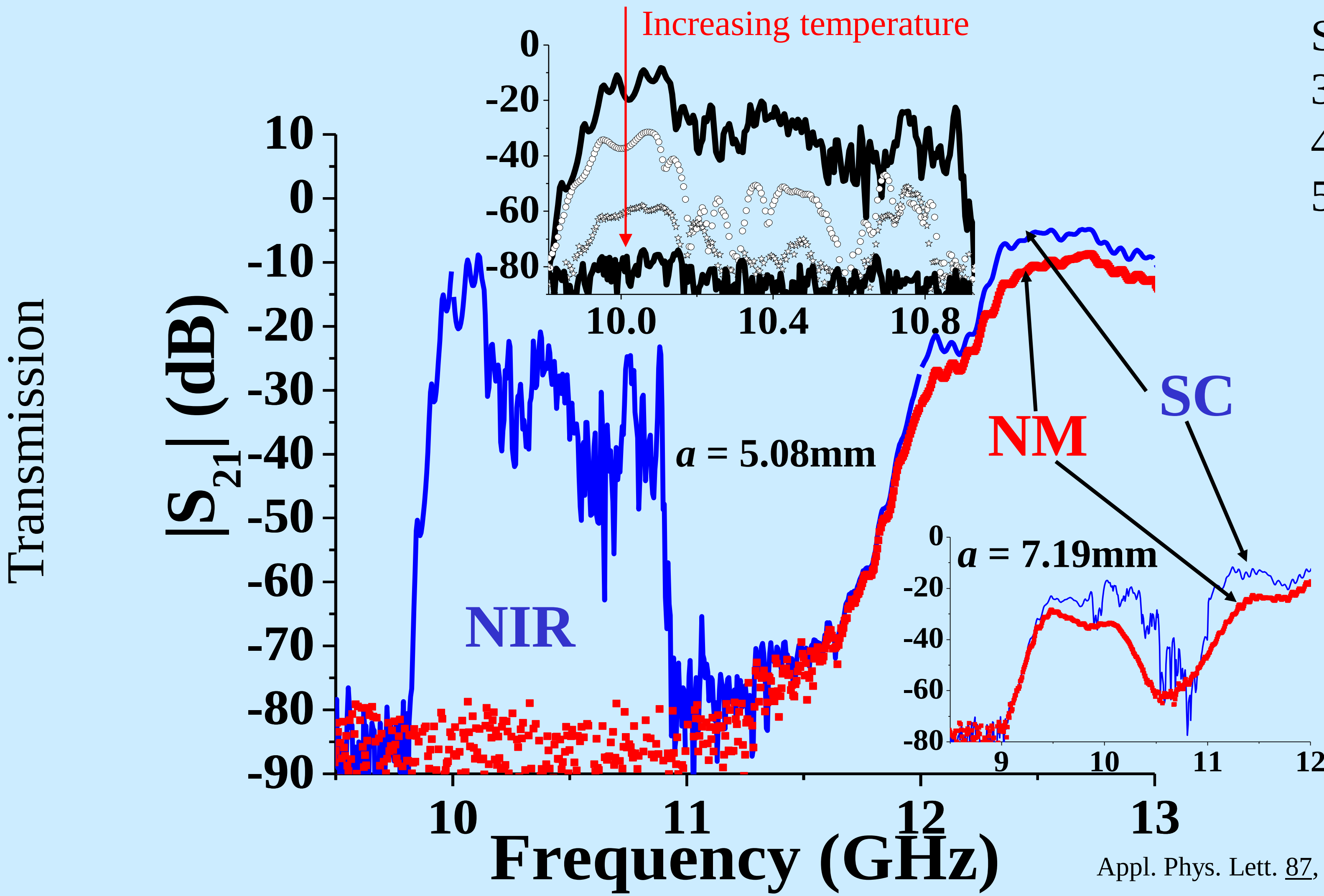


Plasma frequency ~17 GHz





Negative Index Passband with a Superconducting All-Nb Metamaterial



See:
3EZ-01
4EB-05
5EPG-05

Appl. Phys. Lett. 87, 034102 (2005)
arXiv:1004.3226

References and Further Reading

Z. Y. Shen, "High-Temperature Superconducting Microwave Circuits,"
Artech House, Boston, 1994.

M. J. Lancaster, "Passive Microwave Device Applications,"
Cambridge University Press, Cambridge, 1997.

M. A. Hein, "HTS Thin Films at Microwave Frequencies,"
Springer Tracts of Modern Physics 155, Springer, Berlin, 1999.

"Microwave Superconductivity,"
NATO- ASI series, ed. by H. Weinstock and M. Nisenoff, Kluwer, 2001.

T. VanDuzer and C. W. Turner, "Principles of Superconductive Devices and Circuits,"
Elsevier, 1981.

T. P. Orlando and K. A. Delin, "Fundamentals of Applied Superconductivity,"
Addison-Wesley, 1991.

R. E. Matick, "Transmission Lines for Digital and Communication Networks,"
IEEE Press, 1995; Chapter 6.

Alan M. Portis, "Electrodynamcis of High-Temperature Superconductors,"
World Scientific, Singapore, 1993.

Superconductivity Links

Wikipedia article on Superconductivity
http://en.wikipedia.org/wiki/Superconductivity

Superconductor Information for the Beginner
http://www.superconductors.org/

Gallery of Abrikosov Vortex Lattices
http://www.fys.uio.no/super/vortex/

Graduate course on Superconductivity (Anlage)
http://www.physics.umd.edu/courses/Phys798S/anlage/Phys798SAnlageSpring06/index.html

YouTube videos of Superconductivity (Alfred Leitner)
http://www.youtube.com/watch?v=nLWUtUZvOP8

Please Ask Questions!